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# Supply Chain Management of Fresh Products with Producer Transportation\*

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## ABSTRACT

This article considers a class of fresh-product supply chains in which products need to be transported by the upstream producer from a production base to a distant retail market. Due to high perishability a portion of the products being shipped may decay during transportation, and therefore, become unsaleable. We consider a supply chain consisting of a single producer and a single distributor, and investigate two commonly adopted business models: (i) In the "pull" model, the distributor places an order, then the producer determines the shipping quantity, taking into account potential product decay during transportation, and transports the products to the destination market of the distributor; (ii) In the "push" model, the producer ships a batch of products to a distant wholesale market, and then the distributor purchases and resells to end customers. By considering a price-sensitive end-customer demand, we investigate the optimal decisions for supply chain members, including order quantity, shipping quantity, and retail price. Our research shows that both the producer and distributor (and thus the supply chain) will perform better if the pull model is adopted. To improve the supply chain performance, we propose a fixed inventory-plus factor (FIPF) strategy, in which the producer announces a pre-determined inventory-plus factor and the distributor compensates the producer for any surplus inventory that would otherwise be wasted. We show that this strategy is a Pareto improvement over the pull and push models for both parties. Finally, numerical experiments are conducted, which reveal some interesting managerial insights on the comparison between different business models. [Submitted: March, 22, 2011. Revised: September 28, 2011; Accepted: December 22, 2011]

Subject Areas: Long-distance Transportation, Perishable Products, Pull/Push Supply Chains, and Supply Chain Management.

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## INTRODUCTION

It is common practice for firms engaged in the producing, distributing, and retailing of fresh products (e.g., live seafood, fresh fruit, fresh vegetables, cut flowers) to transport their products, either through third-party logistic providers or using their own vehicles, from the production base to distant markets. Due to the highly perishable nature of the products, transportation is regarded as an important link in fresh-product supply chains because producers and/or distributors face the risk that a portion of the products may decay during transportation. As indicated by Wilson, Boyette, and Estes (1995): "Fresh fruits, vegetables, and flowers are highly perishable because they are alive.... They can become sick, deteriorate, and die. Dead fresh fruits and vegetables are not marketable!"

Empirical data show that loss during the distribution chain of fresh products is significant in developed and developing countries. Ferguson and Ketzenberg (2006) noted that grocery retailers in developed Western economies can incur losses of up to 15% due to damage and spoilage of perishable items. An Accenture report showed that in China, the annual loss in fruit and vegetables is around \$8.9 billion, almost 30% of China's annual output (Bolton & Liu, 2006). The discarded portion of fresh products is reported to be high, including 30%-50% for mangos, 20% for bananas, 40%–50% for pineapples, and 30%–50% for oranges (Anonymous, 2003). A recent report from the National Development and Reform Commission (NDRC) reveals that the food spoilage rate is still high in China, with 20% of fresh fruit and vegetables, 30% of fresh meat, and 15% of seafood being spoiled in delivery, costing CNY100 billion a year (Anonymous, 2010). It is recognized that long distance transportation accounts for the largest portion of product losses, especially for countries/regions that lack sophisticated transportation facilities. For example, Leung (2008) indicated that "transport delays and inadequate cold storage cause 30%-40% of fruit and vegetables to rot at the harvesting site or while in transit."

The perishability of products during transportation creates great challenges for companies involved in the supply chain. For example, the weight loss of seafood during transportation is one of the major concerns for fishery companies located in coastal areas of China (e.g., Guangdong and Shandong provinces) in their supply chain management. This is because the product deterioration not only means losses to the companies, but also affects the inventory and pricing decisions of the producer and distributor in the process of matching supply with demand. Intuitively, the allocation of transportation risk (i.e., the risk that some portion of products may decay during transportation) among supply chain members varies across the different ways of doing business between upstream and downstream companies. For example, considering the economies of scale in transportation, some producers prefer to transport their products either by their own vehicles or by third-party logistics providers, to supply distributors/retailers in distant markets; in certain industries, some distributors and large retail chains (e.g., Walmart) prefer to consolidate different goods purchased and conduct the transshipment themselves. Normally, the loss from product deterioration is supposed to be borne by the company that owns the product during transportation. As a result, whether the producer or the distributor is responsible for the transportation may have a different impact on their respective profitability.

A recent work by Cai, Chen, Xiao, and Xu (2010) studied optimal ordering and pricing decisions and coordination mechanisms for a fresh-product supply chain in which the distributor is responsible for the transportation process (i.e., they consider the free on board (FOB) business model). In this article, we will focus on another class of business models in which the transportation is under the responsibility of the upstream producer. In the FOB business model studied by Cai et al. (2010), transportation and market risks (i.e., risk from random fluctuations of demand) are both borne by the downstream distributor. However, in this article, the potential loss from product decay is subject to the producer. Except for market risk, the distributor also faces the risk that the producer may be unable to deliver up to the level expected because the distributor has an unreliable supplier.

As we will show in later sections, there are variants of business models between producers and distributors, with the producer being responsible for transporting the products. For example, there are different practices in the cut flower industry between the southern and northern regions of China. In the south (e.g., Yunnan province) many distributors order the product before the flower supplier ships the products, whereas in the north (e.g., Liaoning province) many producers simply drive the flowers for 5–6 hours to distant wholesale markets (e.g., Beijing) and sell to local distributors. These business model variants also exist in other areas, such as the fishery industry. In Chinese restaurants, many slap-up aquatic products (such as sturgeon, salmon, lobster, and abalone) are usually ordered by the restaurants and then transported by the supplier via a home-delivery or doorstep service; whereas many ordinary breeds (such as grass carp, herring, catfish, and snakehead) are normally purchased directly from local wholesale markets.

Following Cachon (2004), in this article we call the two variants of business models as pull and push scenarios respectively; a detailed description of them will be provided in the next section. Note that Cachon (2004) studies how the allocation of inventory risk (via push, pull, and advance-purchase discount contracts) impacts supply chain efficiency. They do not, however, study the perishability of the product. Therefore, to a certain degree, this article can be viewed as an extension of his work by considering product perishability. From the gaming perspective, to maximize profits in the pull model, the distributor acts as a Stackelberg game leader (and the producer acts as a follower), whereas in the push model the producer acts as a Stackelberg game leader (and the distributor acts as a follower). Normally, it is expected that being a game leader could provide an advantage; in other words, the distributor may prefer the pull model whereas the producer may prefer the push model. Then, if we consider the potential product decay during transportation undertaken by the upstream producer, it is natural to ask: What is the difference between the pull and push models? Or more specifically, how should the supply chain members determine the optimal shipping quantity, order quantity, and pricing decisions in different business models? Why do these variants of business models co-exist in practice? Is it because one of the business models is more beneficial to any of the supply chain members? These are some key issues that may be of interest to companies involved in fresh-product supply chains.

The main purpose of this article is to seek answers to the aforementioned issues. We will focus on a stylized supply chain that consists of a single upstream supplier and a single downstream firm. For convenience, we call the upstream and downstream firms the producer and distributor, respectively. The producer is responsible for transporting the products to a distant market where the distributor is located. Therefore, the producer bears the transportation risk, whereas the risk from market demand is borne by the distributor. Differing on whether the distributor orders *before* or *after* the producer transports the products, we characterize the variants of practices with the following two business models (a more detailed illustration of the models will be given in the next section).

- (i) In the pull model, the flow of inventory is triggered by an order from a downstream distributor. That is, the distributor orders first, then the producer determines the shipping quantity, considering the possible product decay during transportation, and then transports the product to the destination market of the distributor.
- (ii) In the push model, the flow of inventory starts with the producer proactively shipping products. That is, the producer first ships a batch of products to the distant wholesale market, and then the distributor purchases and resells to the retail market.

By considering a price-sensitive end-customer demand, we will first study the optimal decisions for the supply chain members under the two business models, which include the shipping quantity of the producer, and the order quantity and retail price of the distributor. We then provide an in-depth comparison of the optimal performance under the two business models. Based on the managerial insights obtained from the analysis, we will develop modified business models that could help improve the performance of supply chain members.

Our research falls under the field of inventory and supply chain management of perishable products, a topic that has been studied extensively in the literature. Early work on a perishable inventory problem was described by Whitin (1957), where fashion goods deteriorating at the end of certain storage periods were considered. Since then, considerable attention has been focused on this line of research. Nahmias (1982) provides a comprehensive survey of research published before the 1980s. More recent studies on deteriorating inventory models can be found in the surveys of Raafat (1991) and Goyal and Giri (2001), which review relevant literature published in the 1980s and 1990s, respectively. It is widely recognized that the effect of product perishability is two-fold: on the one hand, product quality and value may degrade over time, and on the other hand, the marketable (or surviving) quantity decreases because some portion of the product may be damaged and become unsaleable (e.g., Goyal & Giri, 2001; Blackburn & Scudder, 2009). Of particular relevance to our study are models that deal with quantity losses. In the literature, quantity loss is generally modeled with a probability distribution. For example, Ghare and Schrader (1963) developed an EOQ model for products in which the number of usable units is subject to exponential decay. Covert and Philip (1973) and Philip (1974) used the Weibull distribution to model item deterioration.

Tadikamalla (1978) examined the case of Gamma distributed deterioration. Cai et al. (2010) used a general distribution to characterize the random marketable portion of a batch of products.

Being highly perishable, fresh products create even greater challenges for managers seeking to match supply with demand. Therefore, inventory management and other pertinent management issues of fresh produce have recently attracted the interest of researchers. For example, Zuurbier (1999) investigated factors that influence vertical coordination in the fresh produce industry. Ferguson and Ketzenberg (2006) examined the value of information sharing between retailers and suppliers of fresh products. Ferguson and Koenigsberg (2007) recently presented a two-period model where the quality of the leftover inventory is often perceived to be lower by customers, and the firm can decide to carry all, some, or none of the leftover inventory to the next period. Blackburn and Scudder (2009) examined supply chain design strategies for fresh produce, using melons and sweet corn as examples. Cai et al. (2010) studied the optimization and coordination of fresh product supply chains considering freshness-keeping effort as a decision variable. Focusing on the distribution link of fresh product supply chains, Bolton and Liu (2006) examined the cold supply chain in China from five perspectives: principal challenges, recent developments, new market drivers, key success factors, and implementation considerations. Cattani, Perdikaki, and Marucheck (2007) explored the degree of product perishability's influence on profitability by considering two competing online grocers.

Differing from conventional supply chain models that only consider uncertainties associated with market demand, our model also considers risks that arise from product decay during transportation. As such, both the product supply and demand involve uncertainties, which creates great difficulties in matching supply with demand. In this respect, our work is also related to the body of literature on random yield and/or unreliable suppliers. Yano and Lee (1995) reviewed previous studies of lot-sizing models when production or procurement yields are random. Researchers have used different functions to characterize the reliability of supplies. These include "all-or-nothing delivery" (e.g., Anupindi & Akella, 1993; Gerchak, 1996), random capacity (e.g., Ciarallo, Akella, & Morton, 1994), binomial yield (e.g., Chen, Yao, & Zheng, 2001), stochastic proportional yield (e.g., Henig & Gerchak, 1990), and combinations of these different functions (e.g., Wang & Gerchak, 1996). For comparisons of different models, refer to the recent work by Dada, Petruzzi, and Schwarz (2007), who considered the problem of a newsvendor served by multiple suppliers, where any given supplier may be unreliable. In this article, we adopt the stochastic proportional yield model to characterize the surviving quantities of the products.

The remainder of the article is organized as follows. In the next section we will present the problem descriptions, assumptions, and notation. After that we will derive the optimal decisions for the producer and the distributor in the two business models and conduct a comparative analysis between them. We will then explore extended business models that could improve the performance of the producer and the distributor. Some results from the numerical experiments will be reported before we conclude the article.

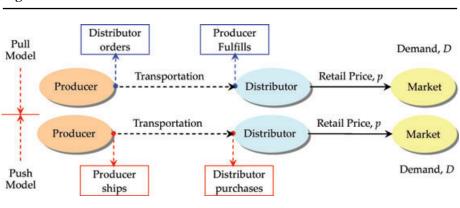


Figure 1: The two business models under consideration.

#### THE MODELS

Like Cai et al. (2010), we consider a supply chain consisting of one producer and one distributor (Figure 1). The distributor purchases from the producer and sells to end customers that are geographically far from the production base; therefore products must undergo long-distance transportation before reaching the market. Due to high perishability, a portion of the products being shipped may decay during the transportation process. That is, the marketable quantity at the destination will be less than or equal to that loaded onto the transportation vehicle. We introduce a random surviving index,  $\Theta$ , defined over [0, 1] to model the perishability of products, with  $\Theta = 1$  and 0 representing, respectively, 100% and 0% of the product surviving when it reaches the market. Note that the realization of  $\Theta$  may be jointly determined by the actual transportation time, the weather condition, the effectiveness of cooling facilities, and other unforeseen factors. Suppose  $\Theta$  follows a continuous distribution, with PDF  $f(\cdot)$ , CDF  $F(\cdot)$ , and mean value  $\mu = \mathbf{E}{\Theta}$  (0  $< \mu < 1$ ).

Let the unit production cost and transportation cost be  $c_1$  and  $c_2$ , respectively; and let the wholesale price charged by the producer be  $w(>c_1 + c_2)$ , which is exogenous. Following Petruzzi and Dada (1999), Wang (2006), and Wang, Jiang, and Shen (2004), we adopt the multiplicative functional-form; in other words, given that the distributor charges a retail price p, the market demand is given by

$$D(p) = y_0 p^{-k} \varepsilon, \quad k > 1,$$

where  $y_0$  is a constant, k is the price elasticity, and  $\varepsilon$  is a random variable representing the random fluctuations of the market demand. To avoid trivial cases, we focus on a price-sensitive market and therefore assume k > 1. Let the PDF and CDF of  $\varepsilon$  be g(x) and G(x), respectively. In addition, we make the following assumption:

Assumption 1: The random factor  $\varepsilon$  has an increasing generalized failure rate: h(x): =xg(x)/[1 - G(x)] is increasing in  $x \in (0, +\infty)$ ; and  $\lim_{x\to\infty} x[1 - G(x)] = 0$ .

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The increasing generalized failure rate (IGFR) condition is a mild restriction on the demand distribution. IGFR is a weaker condition than increasing failure rate (IFR)—a property satisfied by many distributions such as truncated normal, uniform, and the gamma and Weibull families, subject to parameter restrictions (see Lariviere & Porteus, 2001; Mookherjee & Friesz, 2008; and references therein). The condition  $\lim_{x\to\infty} x[1 - G(x)] = 0$  is satisfied by the aforementioned distribution functions.

Consider the scenarios in which the producer is responsible for the transportation process (therefore any loss from product decay will be borne by the producer). With long-distance transportation involved, multiple forms of transaction exist between the producer and the distributor. We consider the following two business models, depending on whether the distributor orders and purchases *before* or *after* the products are transported to the distant market (Figure 1).

- (i) In the pull model, the transaction is similar to the cost insurance and freight scheme that is used widely in foreign trades. In this model, the flow of inventory is triggered by an order from a downstream distributor. That is, the distributor first places an order requesting q units of product. Considering potential product decay during transportation, the producer chooses shipping quantity, Q, which may be greater than that ordered by the distributor, and transports it, either by its own vehicle or by third-party logistics providers, to the distant market designated by the distributor. There are two possible outcomes after transportation: (a) If the surviving quantity of the producer is no less than q, then the distributor obtains all the product that he has ordered; (b) otherwise, the producer will be unable to fulfill the entire order of the distributor, and the maximal quantity that the distributor can obtain is constrained by the surviving products. In both cases, after the transaction between the two parties is conducted, the distributor sets a retail price, denoted by p, for sales to end customers.
- (ii) In the push model, the flow of inventory starts with the producer proactively shipping products, "pushing" them from upstream to downstream along the supply chain. That is, the producer first determines the quantity (Q) to be shipped and transports the products to a distant wholesale market, where the distributor decides on the purchase quantity (q) based on a prenegotiated wholesale price (w), with the maximal quantity constrained by the producer's surviving quantity. Meanwhile, the distributor sets a retail price, denoted by p, for sales to end customers.

In summary, the distributor orders before transportation in the pull model, whereas orders are done after transportation in the push model. We have the following assumption.

Assumption 2: The wholesale price of the producer is greater than  $\frac{c_1+c_2}{\mu}$  (i.e.,  $w > \frac{c_1+c_2}{\mu}$ ).

To illustrate the reasoning behind this assumption, suppose the producer ships one unit of product in either business model. Note that the mean value of  $\Theta$  can be interpreted as the probability that the unit of product will survive, and

 $(1 - \mu)$  is the probability that it becomes unsaleable. Therefore, the producer's expected revenue from selling this unit of product is given by  $\mu w$ , and the cost is  $c_1 + c_2$ . We need the condition presented in Assumption 2 to ensure that the producer is willing to ship at least one unit of product; otherwise, the transaction between the producer and the distributor will be uneconomic for the producer.

For both business models, the optimal decisions of each party are made by considering the best response of the other party. To facilitate the characterization of optimal decisions, we assume that all information is common knowledge to both supply chain members. Following the convention in the literature, both parties are assumed to be risk-neutral; they seek to maximize their respective expected profit. Finally, to simplify the model, we do not consider any salvage value of the products left unsold (recall that the product is highly perishable). That is, even if the actual delivery amount is larger than the purchase quantity requested by the distributor, the producer generates a zero revenue from the surplus inventories because she cannot sell them to the end-market; and after all the end-market demands are realized, the distributor obtains a zero salvage value from any remaining inventories as well.

Throughout the article, we use subscript "c" and "s" in the decision variables, and superscript "c" and "s" in the profit functions to denote the pull model and push model, respectively.

## **OPTIMAL DECISIONS**

Before investigating the respective optimal decisions for the two business models, it should be noted that in both models, the distributor eventually faces the problem of setting an optimal retail price (p), which may depend on the on-hand inventory level. Because the purchasing cost paid to the producer is regarded as sunk, the distributor needs to optimize the retail price from maximizing his expected selling revenue. Suppose the distributor's marketable quantity is q, thus his expected selling revenue as a function of the retail price p is

$$R_d(p \mid q) = \mathbf{E}\{p \min(D(p), q)\} = p\mathbf{E}\{\min(y_0 p^{-k}\varepsilon, q)\}.$$

The optimal retail price is presented in the following Lemma.

*Lemma 1*: Given that the distributor's on-hand inventory level is q, the optimal retail price, which is a function of q, should be set at

$$p^*(q) = \left(\frac{z_0 y_0}{q}\right)^{1/k},$$
 (1)

where  $z_0$  is uniquely determined by

$$(k-1)\int_0^z xg(x)dx = z[1-G(z)].$$
 (2)

**Proof:** Following Petruzzi and Dada (1999), we define  $z := q/[y_0p^{-k}]$  and call it the stocking factor. Then the distributor's revenue function  $R_d(p | q)$  can be rewritten as

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$$R_d(z \mid q) = (zy_0)^{1/k} q^{1-1/k} \left( 1 - \int_0^z (1 - x/z)g(x) \mathrm{d}x \right).$$

The optimal stocking factor that maximizes  $R_d(z | q)$  must satisfy the following first-order condition:

$$\frac{dR_d(z \mid q)}{dz} = \frac{y_0^{1/k} q^{1-1/k}}{z^{1-1/k} k} \left( 1 - \int_0^z \left[ \frac{x}{z} (k-1) + 1 \right] g(x) dx \right) = 0;$$

from this we can show that the optimal stocking factor  $z_0$  must satisfy Equation (2). We next prove the uniqueness of  $z_0$ . Let

$$\phi(z) := \int_0^z [x(k-1) + z]g(x)dx - z = -z\bar{G}(z) + (k-1)\int_0^z xg(x)dx,$$

where  $\overline{G}(z) := 1 - G(z)$ . Then we have

$$\phi'(z) = \int_0^z g(x) dx + zkg(z) - 1 = zkg(z) - \bar{G}(z) = k\bar{G}(z) \left[ h(z) - \frac{1}{k} \right].$$

Note that by Assumption 1, the generalized increasing failure rate function of  $\varepsilon$ , h(x) is increasing, therefore  $\phi(z)$  decreases before z reaches  $h^{-1}(1/k)$  and increases after  $h^{-1}(1/k)$ , and hence is unimodal. As  $\phi(0) = 0$  and  $\lim_{z\to\infty}\phi(z) > 0$ , it is apparent that  $\phi(z) = 0$  has only one solution within  $(0, \infty)$ ; therefore,  $z_0$ is uniquely determined by Equation (2). It's trivial that for  $z > z_0$ ,  $\phi(z) > 0$  and thus  $R'_d(z | q) < 0$ ; for  $z < z_0$ ,  $\phi(z) < 0$  and thus  $R'_d(z | q) > 0$ . Therefore,  $R_d(z | q)$ is unimodal in z, and  $z_0$  is the unique maximizer of  $R_d(z | q)$ . This completes the proof.

Lemma 1 gives a closed-form solution for the optimal retail price. From Equation (1) we know that the distributor should decrease the retail price when there is more inventory; this is consistent with our intuition. Substituting Equation (1) into  $R_d(p | q)$ , we obtain the optimal retail revenue as

$$R_d^*(q) := R_d(p^*(q) | q) = \frac{k}{k-1} \bar{G}(z_0) (z_0 y_0)^{1/k} q^{1-1/k} := \frac{k}{k-1} A q^{1-1/k},$$
(3)

where for notational simplicity, we let the constant  $A = \overline{G}(z_0)(z_0y_0)^{1/k}$ .

#### **Optimal Decisions for the Pull Model**

We summarize the sequence of key events that occur in the pull model as follows. (i) The distributor determines the order quantity  $q_c$ ; (ii) the producer determines the shipping quantity  $Q_c$  and transports the products; (iii) the distributor receives the products and determines the retail price  $p_c$ ; and (iv) customer demand is realized and satisfied. Note that Lemma 1 already presents the optimal retail pricing decision; we will solve the other decision problems for the two parties in backwards order.

First, given that the producer receives an order quantity of  $q_c$  units from the distributor, the producer may have to ship more than  $q_c$  units of product, because some portion of the product may decay before it arrives at the distant market. On one hand, the producer has to ship more inventory to avoid any supply shortage; on the other hand, over-shipping is also less desirable, because limited revenues

will be realized, and will incur a loss to the producer. The trade-off between the benefits and costs of over-shipping must be managed to maximize the producer's expected profit.

The expected profit function of the producer with respect to the shipping quantity  $Q_c$  is as follows:

$$\Pi_{p}^{c}(Q_{c} | q_{c}) = \mathbf{E}\{w \min(q_{c}, Q_{c}\Theta) - (c_{1} + c_{2})Q_{c}\} = wq_{c} - (c_{1} + c_{2})Q_{c} - w \int_{0}^{q_{c}/Q_{c}} (q_{c} - Q_{c}x)f(x)dx.$$
(4)

**Theorem 1:** In the pull model, given that the distributor orders  $q_c$ , the producer's optimal shipping quantity is given by  $Q_c^*(q_c) = q_c/\theta_c$ , where  $\theta_c$  equals  $\theta_0$ , the unique solution for the following equation:

$$\int_{0}^{\theta_{0}} x f(x) \mathrm{d}x = \frac{c_{1} + c_{2}}{w}.$$
(5)

**Proof:** The first derivative of  $\prod_{d=1}^{c} (Q_c \mid q_c)$  is

$$\frac{\mathrm{d}\Pi_p^c(Q_c \,|\, q_c)}{\mathrm{d}Q_c} = w \int_0^{q_c/Q_c} x f(x) \mathrm{d}x - (c_1 + c_2),$$

which is decreasing in  $Q_c$ . Therefore,  $\prod_p^c (Q_c | q_c)$  is concave, and the optimal shipping quantity should be determined by the first order condition, from which we have  $Q_c^*(q_c) = q_c/\theta_c$ , where  $\theta_c$  solves Equation (5). Note that for  $\forall \theta > 0$ ,  $0 < \int_0^{\theta} xf(x)dx \le \mathbf{E}\{\Theta\} = \mu$ . By Assumption 2, we know that the right-hand side of Equation (5),  $\frac{c_1+c_2}{w} < \mu$ ; therefore the solution to Equation (5) exists in (0, 1) and is unique. This completes the proof.

Note that we have  $0 < \theta_c < 1$ , therefore  $1/\theta_c$  can be regarded as an "inventoryplus" factor and  $(1/\theta_c - 1)q_c$  is the extra quantity added to cater for the transportation risk. The "service level" of the producer, defined as the probability that the distributor's entire order is satisfied, is given by

$$\Pr\{Q_c^*(q_c)\Theta \ge q_c\} = \Pr\left\{\frac{q_c}{\theta_c}\Theta \ge q_c\right\} = \Pr\{\Theta \ge \theta_c\} = \bar{F}(\theta_c).$$
(6)

Theorem 1 shows that  $\theta_c$  is decreasing in w; this implies the intuitive result that when the opportunity cost from supply shortage is high, the distributor should increase the inventory-plus factor and service level to avoid losses that may arise from supply shortage. Moreover, Theorem 1 illustrates the reasonableness of Assumption 2: the first derivative of  $\Pi_d^c(Q_c | q_c)$  is always non-positive if  $w \leq \frac{c_1+c_2}{\mu}$  (i.e.,  $\Pi_p^c(Q_c | q_c)$  is non-increasing in  $Q_c$ ) and therefore the optimal shipping decision is  $Q_c^* = 0$ .

Substituting Equation (5) into Equation (4), the producer's optimal profit for a given ordering quantity  $q_c$  is

$$\Pi_{p}^{c}(Q_{c}^{*}(q_{c}) | q_{c}) = wq_{c}[1 - F(\theta_{c})],$$
(7)

which is proportional to the distributor's ordering quantity (note that  $\theta_c$  is independent of  $q_c$ ). Because  $1 - F(\theta_c) > 0$ , the producer's expected profit will always be positive. Therefore, as expected, the producer always prefers a larger order quantity from the distributor.

Knowing that the producer will ship  $1/\theta_c$  times the quantity being ordered, the distributor determines his order quantity, considering the possible amount that may be received (which is constrained by the producer's surviving quantity  $Q_c^*(q_c)\Theta$ ). It is readily shown that only when  $\Theta \ge \theta_c$  (with probability  $1 - F(\theta_c)$ ), could the producer fulfill the distributor's entire order. By incorporating the optimal retail price decision (Lemma 1) and the corresponding retail revenue (Equation (3)), and conditioning on the random surviving factor  $\Theta$ , we write the distributor's expected profit as follows:

$$\Pi_{d}^{c}(q_{c}) = \mathbf{E} \{ R_{d}^{*}(\min(q_{c}, Q_{c}^{*}(q_{c})\Theta)) - wq_{c} + w(q_{c} - Q_{c}^{*}(q_{c})\Theta)^{+} \}$$

$$= -wq_{c} + \frac{k}{k-1} Aq_{c}^{1-1/k} \bar{F}(\theta_{c})$$

$$+ \int_{0}^{\theta_{c}} \left[ \frac{kA}{k-1} \left( \frac{q_{c}}{\theta_{c}} x \right)^{1-1/k} + wq_{c} \left( 1 - \frac{x}{\theta_{c}} \right) \right] f(x) \mathrm{d}x, \qquad (8)$$

where  $x^+ := \max(x, 0)$ , and the three items on the right-hand side of the first line correspond to the distributor's retail revenue, the wholesale cost paid to the producer, and the wholesale refund from the producer, respectively.

**Theorem 2:** In the pull model, the distributor's expected profit function is concave, and his optimal order quantity is

$$q_c^* = \left(\frac{A}{w}\Omega(\theta_c)\right)^k,\tag{9}$$

where function  $\Omega(\cdot)$  is defined as

$$\Omega(\theta) := \frac{\int_0^{\theta} \left(\frac{x}{\theta}\right)^{1-1/k} f(x) dx + \bar{F}(\theta)}{\int_0^{\theta} \frac{x}{\theta} f(x) dx + \bar{F}(\theta)}$$

**Proof:** Taking the first derivative of  $\Pi_d^c(q_c)$  with respect to  $q_c$ , we have

$$\frac{\mathrm{d}\Pi_d^c(q_c)}{\mathrm{d}q_c} = -w + A\bar{F}(\theta_c)q_c^{-1/k} + \int_0^{\theta_c} \left[A\left(\frac{x}{\theta_c}\right)^{1-1/k}q_c^{-1/k} + w\left(1 - \frac{x}{\theta_c}\right)\right]f(x)\mathrm{d}x,$$

which is decreasing in  $q_c$  (recall that we have assumed k > 1). Therefore,  $\Pi_d^c(q_c)$  is concave and has a unique maximum. By applying the first order condition, we arrive at Equation (9). This completes the proof.

Theorem 2 implies that the distributor's order quantity is mainly determined by function  $\Omega(\theta_c)$ . Because k > 1, and  $\theta_c > 0$ , it is not difficult to show that

$$\Omega(\theta_c) := \frac{\int_0^{\theta_c} \left(\frac{x}{\theta_c}\right)^{1-1/k} f(x) dx + \bar{F}(\theta_c)}{\int_0^{\theta_c} \frac{x}{\theta_c} f(x) dx + \bar{F}(\theta_c)}$$
$$= 1 + \frac{\int_0^{\theta_c} \left[\left(\frac{x}{\theta_c}\right)^{1-1/k} - \frac{x}{\theta_c}\right] f(x) dx}{\int_0^{\theta_c} \frac{x}{\theta_c} f(x) dx + \bar{F}(\theta_c)} > 1.$$

As a result, we know that

$$q_c^* > \left(A/w\right)^k,\tag{10}$$

where the right-hand side can be readily shown to be the optimal order quantity when the product is non-perishable during transportation (i.e., when the supply is reliable). Therefore, the perishability of products tends to motivate the distributor to order more. Moreover, it is not difficult to show that function  $\Omega(\theta)$  is increasing in  $\theta$ . Therefore Equation (9) implies that the distributor tends to order more knowing the producer will choose a smaller inventory-plus factor  $(\frac{1}{\theta})$  because the supply becomes less reliable.

By considering the distributor's optimal order quantity (Theorem 2), we summarize the optimal performance for the two supply chain members in the pull model. The producer's optimal profit is

$$\Pi_{p}^{c*} := \Pi_{p}^{c}(Q_{c}^{*}(q_{c}^{*}) | q_{c}^{*}) = A^{k} w^{1-k} \Omega^{k}(\theta_{c}) \left[1 - F(\theta_{c})\right];$$
(11)

and the distributor's optimal profit is

$$\Pi_{d}^{c*} := \Pi_{d}^{c}(q_{c}^{*}) = \frac{1}{k-1} \left(\frac{A\Omega(\theta_{c})}{w}\right)^{k} \left[w \left[1 - F(\theta_{c})\right] + \frac{c_{1} + c_{2}}{\theta_{c}}\right].$$
(12)

From Equations (11) and (12), we have the following ratio:

$$\frac{\Pi_d^{c*}}{\Pi_p^{c*}} = \frac{1}{k-1} \left[ 1 + \frac{\int_0^{\theta_c} x f(x) \mathrm{d}x}{\theta_c \bar{F}(\theta_c)} \right],$$

which is clearly decreasing in k. This implies that the distributor will obtain a larger portion of the entire-chain profit when market demand is less price-sensitive (i.e., when k is small). To illustrate, consider an extreme case in which demand is almost insensitive to a change in price (i.e.,  $k \rightarrow 1$ ). Then naturally the distributor could set a sufficiently high retail price without affecting the magnitude of market demand. Thus, most supply-chain profit is obtained by the distributor, because the unit profit of the producer is limited (note that her unit profit is  $w - c_1 - c_2$ ).

### **Optimal Decisions for the Push Model**

In the push model, the sequence of events is as follows. (i) The producer determines the shipping quantity  $Q_s$  and transports it to the distant wholesale market; (ii) the distributor determines the joint decisions on purchasing quantity  $q_s$  and retail price  $p_s$ , considering the producer's available supply; and (iii) customer demand is realized and satisfied. We solve the decision problems for the two parties in reverse order.

Firstly, given that the producer has shipped  $Q_s$  units of products and the realized surviving factor after transportation is  $\theta$ , the distributor jointly determines his purchasing quantity (at wholesale price w) and retail price, with the objective of maximizing his expected profit. Putting aside the capacity constraint, we first investigate the distributor's profit function, which is formulated as

$$\Pi_d^s(q_s, p_s) = \mathbf{E} \{ p_s \min(q_s, D(p_s)) - wq_s \}$$
  
=  $-wq_s + p_s \mathbf{E} \{ \min(q_s, y_0 p_s^{-k} \varepsilon) \}.$  (13)

From Lemma 1, we know that for any optimal solution that maximizes  $\Pi_d^s(q_s, p_s)$ , the optimal retail price must be  $p_s^* = p^*(q_s)$ . Therefore the distributor's profit can be rewritten into a form that only depends on  $q_s$ :

$$\Pi_d^s(q_s) := \Pi_d^s(q_s, p^*(q_s)) = -wq_s + \frac{k}{k-1}Aq_s^{1-1/k}.$$
 (14)

By taking the first and second derivatives, we can easily show that  $\Pi_d^s(q_s)$  is concave and its maximal value is achieved at  $q_s^* = (A/w)^k$ . Incorporating the producer's capacity, we immediately arrive at the following theorem.

**Theorem 3:** In the push model, given that the producer's marketable quantity is  $Q_s\theta$ , the distributor's eventual purchasing quantity is min  $((A/w)^k, Q_s\theta)$ .

Knowing that the distributor will eventually order up to  $(A/w)^k$ , the producer seeks to maximize her profit by choosing an appropriate shipping quantity. The producer's expected profit function is

$$\Pi_{p}^{s}(Q_{s}) = \mathbf{E} \{ w \min \left( A^{k} w^{-k}, Q_{s} \Theta \right) - (c_{1} + c_{2}) Q_{s} \}$$
  
=  $-(c_{1} + c_{2}) Q_{s} + A^{k} w^{1-k} \bar{F} \left( A^{k} w^{-k} / Q_{s} \right) + w \int_{0}^{A^{k} w^{-k} / Q_{s}} Q_{s} x f(x) dx.$  (15)

**Theorem 4:** In the push model, the producer's optimal shipping quantity is  $Q_s^* = (\frac{A}{w})^k / \theta_s$ , where  $\theta_s$  equals  $\theta_0$ , the unique solution for Equation (5).

**Proof:** The first derivative of  $\Pi_p^s(Q_s)$  is

$$\frac{\mathrm{d}\Pi_p^s(Q_s)}{\mathrm{d}Q_s} = -(c_1 + c_2) + w \int_0^{A^k w^{-k}/Q_s} x f(x) \mathrm{d}x,$$

which is decreasing in  $Q_s$ . Therefore,  $\prod_p^s(Q_s)$  is concave; the optimal shipping quantity should be determined by the first-order condition, from which we have  $Q_s^* = (\frac{A}{w})^k / \theta_s$ , where  $\theta_s$  solves Equation (5). This completes the proof.

From Theorem 3, when the wholesale price offered by the producer is high, the distributor will purchase less. On the other hand, Theorem 4 shows that the "inventory-plus" factor (over the maximal quantity that the distributor purchases) in the push model,  $1/\theta_s$ , is increasing in w. Therefore, what is the overall impact of the wholesale price on the producer's optimal shipping quantity? To answer this question, we first take derivatives with respect to w on both sides of Equation (5) and arrive at the following:

$$\frac{\mathrm{d}\theta_s}{\mathrm{d}w} = \frac{\mathrm{d}\theta_0}{\mathrm{d}w} = -\frac{c_1 + c_2}{w^2} \times \frac{1}{\theta_s f(\theta_s)} = -\frac{1}{w\theta_s f(\theta_s)} \int_0^{\theta_s} x f(x) \mathrm{d}x.$$

Therefore we have,

$$\begin{aligned} \frac{\mathrm{d}\varrho_s^*}{\mathrm{d}w} &= -\frac{kA^k w^{-k-1}}{\theta_s} - \frac{A^k w^{-k}}{\theta_s^2} \times \frac{\mathrm{d}\theta_s}{\mathrm{d}w} \\ &= -\frac{A^k w^{-k-1}}{\theta_s} \left\{ k - \int_0^{\theta_s} xf(x) \mathrm{d}x \middle/ \theta_s f(\theta_s) \right\}. \end{aligned}$$

Note that  $\theta_s$  is independent of the price elasticity, k. The above equation shows that the relationship between  $Q_s^*$  and w depends on the value of k: when k is large (i.e.,  $k > \int_0^{\theta_s} xf(x)dx/\theta_s f(\theta_s)$ ),  $Q_s^*$  is decreasing in w; on the other hand, when k is small (i.e.,  $k < \int_0^{\theta_s} xf(x)dx/\theta_s f(\theta_s)$ ),  $Q_s^*$  is increasing in w. This is because when demand is more price-sensitive, the maximal quantity the distributor is willing to purchase decreases more steeply in w; whereas the inventory-plus factor is less sensitive to changes.

Next, we summarize the optimal profits for the supply chain members in the push model. By substituting Theorem 4 into Equation (15), we obtain the optimal performance for the producer as follows:

$$\Pi_{p}^{s*} = \Pi_{p}^{s}(Q_{s}^{*}) = -(c_{1} + c_{2})\frac{A^{k}w^{-k}}{\theta_{s}} + A^{k}w^{1-k}\bar{F}(\theta_{s}) + w\int_{0}^{\theta_{s}}\frac{A^{k}w^{-k}}{\theta_{s}}xf(x)dx$$

$$= A^{k}w^{1-k}\bar{F}(\theta_{s}).$$
(16)

Conditioning upon the surviving factor  $\Theta$ , we arrive at the expected profit for the distributor:

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$$\Pi_{d}^{s*} = \int_{0}^{\theta_{s}} \Pi_{d}^{s}(Q_{s}^{*}x)f(x)dx + \int_{0}^{\theta_{s}} \Pi_{d}^{s}(A^{k}w^{-k})f(x)dx$$
$$= \frac{1}{k-1}A^{k}w^{1-k}\bar{F}(\theta_{s}) + \frac{A^{k}w^{1-k}}{\theta_{s}}\int_{0}^{\theta_{s}} \left[-x + \frac{k}{k-1}\theta_{s}^{1/k}x^{1-1/k}\right]f(x)dx.$$
(17)

Note that in the conventional newsvendor problem where supply is 100% reliable, the optimal profit of the downstream distributor is always lower if the upstream supplier charges a higher wholesale price. However, as Equation (17) shows, the distributor's optimal profit in the push model may not necessarily be strictly decreasing in w; this is because a lower w may induce the producer to ship less product (recall that the inventory-plus factor  $\frac{1}{\theta_s}$  is increasing in w) and as a result, the distributor faces an even more unreliable supplier.

#### **Pull versus Push Models**

Having obtained the optimal decisions for the pull and push models in the previous subsections, we now conduct a comparison between the two models. We first remark that if the product is not perishable during transportation, then the optimal decisions and optimal expected profit under the pull and push models will be exactly the same, because the producer will always choose an inventory-plus factor of 1. Therefore, the differences between the two models can be attributed to the possibility of product decay during the distribution chain.

First, Theorems 1 and 4 show that the producer will choose the same inventory-plus factor under the two models (recall that  $\theta_c = \theta_s = \theta_0$ ). This is because the choice of inventory-plus factors is based on the trade-off between the shortage cost (w) and over-stocking cost ( $c_1 + c_2$ ). Given that the parameters are assumed the same for the two models, the decision is independent of how much the distributor orders.

Nevertheless, as Theorems 2 and 3 show, the distributor may choose a different order quantity for the different models. We have shown that  $\Omega(\theta_0) > 1$ , therefore clearly we have

$$q_c^* > q_s^*$$
 and  $Q_c^* > Q_s^*$ .

This means that in the pull model, the distributor will order more and therefore the producer will ship more products. To illustrate the reasoning behind this, we consider the distributor's expected profit as a function of the producer's available inventory for the two models (Figure 2). Note that the distributor's unconstrained expected profit function is concave (as the dashed lines show), with the maximal value being achieved at  $(A/w)^k$ ; therefore, the bold line in Figure 2(a) characterizes the distributor's profit corresponding to any available inventory level for the *push* model in which the distributor will order up to  $(A/w)^k$ . Due to the unreliability of supplies, the expected profit should always be less than the maximal value of the profit curve.

Next, consider the pull model in which the distributor reveals his order quantity before the producer makes the shipment. This time, the distributor gains some power in influencing the producer's shipping quantity. By increasing the order quantity from  $(A/w)^k$  to  $(A\Omega(\theta_0)/w)^k$ , the distributor shapes his profit curve

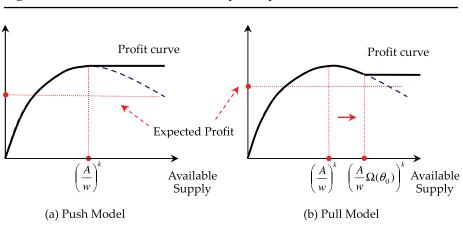


Figure 2: Illustration of distributor's expected profits.

into the form in Figure 2(b). As was shown, the distributor will obtain an even lower profit (than the maximal achievable profit) if the producer satisfies his entire order. However, the overall expected profit *might* be increased.

**Proposition 1:** Compared with the push model, both the producer and the distributor will achieve a higher expected profit if they choose the pull model. That is, we have  $\Pi_p^{c*} > \Pi_p^{s*}$  and  $\Pi_d^{c*} > \Pi_d^{s*}$ .

**Proof:** Note that  $\theta_c = \theta_s = \theta_0$  and  $\Omega(\theta_0) > 1$ ; from Equations (11) and (16) we clearly have  $\Pi_p^{c*} > \Pi_p^{s*}$ . To compare  $\Pi_d^{c*}$  with  $\Pi_d^{s*}$ , we first make some transformations on  $\Pi_d^{s*}$ . By the definition of  $\Omega(\theta)$  and  $\theta_0$ , we have

$$\int_0^{\theta_0} \left(\frac{x}{\theta_0}\right)^{1-1/k} f(x) \mathrm{d}x = [\Omega(\theta_0) - 1]\bar{F}(\theta_0) + \Omega(\theta_0) \frac{c_1 + c_2}{\theta_0 w};$$

substituting this into Equation (17), yields

$$\Pi_{d}^{s*} = \frac{1}{k-1} A^{k} w^{1-k} \bar{F}(\theta_{s}) + \frac{A^{k} w^{1-k}}{\theta_{s}} \int_{0}^{\theta_{s}} \left[ -x + \frac{k}{k-1} \theta_{s}^{1/k} x^{1-1/k} \right] f(x) dx$$
$$= \frac{1}{k-1} \left( \frac{A}{w} \right)^{k} (k \Omega(\theta_{0}) - k + 1) \left[ w \bar{F}(\theta_{0}) + \frac{c_{1} + c_{2}}{\theta_{0}} \right].$$

Therefore, to show  $\Pi_d^{c*} > \Pi_d^{s*}$ , we need only prove:

$$\Omega^k(\theta_0) > k\Omega(\theta_0) - k + 1.$$
(18)

Define function

$$Y(\omega) := \omega^k - k\omega + k - 1, \quad \omega \ge 1,$$

whose first derivative is  $Y'(\omega) = k\omega^{k-1} - k$ . Therefore,  $Y'(\omega)$  is positive for  $\forall \omega > 1$  (recall k > 1). As a result,  $Y(\omega)$  strictly increases within the interval  $[1, +\infty)$ .

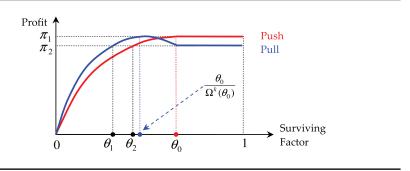


Figure 3: Distributor's expected profit as a function of the surviving factor.

Therefore,  $Y(\omega) > Y(1) = 0$  for  $\forall \omega > 1$ ; this means that inequality (18) holds (because  $\Omega(\theta_0) > 1$ ). This completes the proof.

Therefore, compared with the push model, the distributor benefits from increasing his order quantity in the pull model. We plot the distributor's profits as a function of the producer's realized surviving factor in Figure 3 for the two business models.

As can be seen, although the distributor has greater likelihood of achieving the maximal obtainable profit (denoted as  $\pi_1$ ) in the push model (in the pull model, the probability of obtaining profit  $\pi_1$  is zero), he has an even larger probability of realizing the sub-maximal profit (denoted as  $\pi_2$ ) in the pull model (note that the probability of achieving at least  $\pi_2$  profit are  $\overline{F}(\theta_2)$  and  $\overline{F}(\theta_1)$  for the push and pull models, respectively). Therefore, it seems that the distributor, who acts as a Stackelberg game leader in the pull model, has a more conservative attitude towards risk.

Recall that the above results are obtained by considering a price-dependent market. For certain fresh products, the retail price might be fairly rigid, for example, when it is regulated by the government. Interestingly, we find that when the market demand is price-independent (i.e., when the retail price is exogenous), the major findings still hold. That is, the producer will still ship more products in the pull model, and as a result, the producer and distributor are both better off.

#### IMPROVING SUPPLY CHAIN PERFORMANCE

As demonstrated, in either business model, when the producer's surviving quantity is greater than the amount that has been or will be ordered by the distributor at wholesale price w, the surplus inventory means a loss for the producer because it collects hardly any revenue for the producer (recall that we have assumed zero salvage value); instead, it consumes both production and transportation costs. Clearly, the producer will be better off if the products could be sold at any positive price. On the distributor's side, if he compensates the producer for any redundant inventory, for example, by buying the surplus inventory at a reasonably low price, he might be able to increase profit as well. Thus, the natural question is whether the two supply chain members could increase their respective profits by coming to a compensation agreement on the surplus inventory, and if so, how should the producer price the surplus inventory.

We will answer these questions and explore the possibilities of improving the supply chain performance by developing appropriate contracts in this section. The rest of this section is organized as follows. In the first subsection, we explore the profit impact from offering a secondary transaction after the primary transaction between the distributor and producer has already been conducted. We show the so-called "compensation contract" is beneficial to both parties; this motivates us to study an extended business model with compensation in the second subsection. Unfortunately, when the distributor can expect a secondary transaction opportunity, he may order quite a lot less in the primary transaction and therefore harm the benefit of the producer. As a result, the proposed extended model may not be acceptable to both parties. Therefore, in the third subsection we propose and investigate another strategy called "fixed inventory-plus factor," which is shown to be incentive compatible and a Pareto improvement.

## A Compensation Contract in the Second Period

Suppose when the products arrive at the distant market, the total marketable quantity is  $Q\theta$ . Now the producer has fulfilled the distributor's order of q units (at wholesale price w) and has  $Q\theta - q > 0$  units of surplus inventory. The producer needs to determine a unit compensation rate,  $\tilde{w}$ , for any surplus inventory; she will then offer to sell the  $(Q\theta - q)$  units of products to the distributor at a possibly discounted price  $\tilde{w}$ .

We first investigate the distributor's response to the opportunity to acquire surplus inventory. Suppose the distributor is willing to purchase  $\tilde{q}$  more products at compensation rate  $\tilde{w}$ . By having total quantity of  $q + \tilde{q}$  units for resale to end customers, his expected retail revenue is  $R_d^*(q + \tilde{q})$ ; therefore, the distributor's profit as a function of  $\tilde{q}$  is

$$R_d^*(q+\tilde{q}) - wq - \tilde{w}\tilde{q} = \frac{k}{k-1}A(q+\tilde{q})^{1-1/k} - wq - \tilde{w}\tilde{q},$$

which is readily shown to be concave in  $\tilde{q}$ , and the optimal  $\tilde{q}$  that maximizes the above profit is given by

$$\tilde{q}^* = \max\left(0, \left(\frac{A}{\tilde{w}}\right)^k - q\right);$$

that is, only when the compensation rate offered by the producer is lower than  $Aq^{-1/k}$ , is the distributor willing to purchase more inventory; and by purchasing  $\tilde{q}^*$  more units at  $\tilde{w}$ , the distributor actually increases his profit.

Next, knowing that the distributor will purchase  $\tilde{q}^*$  units, the producer chooses a best compensation rate to maximize her added revenue, which is formulated as:

$$\tilde{w}\min(\tilde{q}^*, Q\theta - q) = \begin{cases} \tilde{w}(Q\theta - q) & \text{if } \tilde{w} \le A(Q\theta)^{-1/k}; \\ A^k \tilde{w}^{1-k} - q \tilde{w} & \text{if } A(Q\theta)^{-1/k} \le \tilde{w} \le Aq^{-1/k}; \\ 0 & \text{if } \tilde{w} \ge Aq^{-1/k}. \end{cases}$$

	Distributor	Producer
Without ST	$\tfrac{k}{k-1}Aq^{1-1/k} - wq$	wq
With ST	$rac{1}{k-1}A(Q heta)^{1-1/k} \ + Aq(Q heta)^{-1/k} - wq$	$wq + A(Q\theta)^{1-1/k} - Aq(Q\theta)^{-1/k}$
Profit improvement	$\frac{\frac{1}{k-1}A(Q\theta)^{1-1/k}}{+Aq(Q\theta)^{-1/k}-\frac{k}{k-1}Aq^{1-1/k}}$	$A(Q heta)^{-1/k}[Q heta-q]$

Table 1: Benefit from compensation in the presence of surplus inventories.

This revenue function increases in the interval  $[0, A(Q\theta)^{-1/k}]$  and decreases in the interval  $[A(Q\theta)^{-1/k}, +\infty]$ ; in other words, the function is unimodal and has a unique maximizer. The producer's optimal compensation rate should be set at

$$\tilde{w}^*(Q\theta) = A(Q\theta)^{-1/k},$$

at which point the distributor is willing to purchase all surplus inventory. It is interesting to note that the optimal compensation rate only depends on the total marketable quantity of the producer, whereas it is independent of the quantity ordered by the distributor. Recall that the distributor's order quantities in the pull and push models are both no less than  $(A/w)^k$ . Therefore, we have

$$\tilde{w}^*(Q\theta) = A(Q\theta)^{-1/k} < Aq^{-1/k} \le w;$$

in other words, the unit compensation rate is a discount over the original wholesale price w.

We summarize the corresponding profits of the supply chain members for the scenarios with/without a compensation contract in Table 1. As expected, both parties will increase their respective profits from the offering of surplus inventory at a discounted price. Note that the conclusion is drawn for any given order quantity q and shipping quantity Q. What if the distributor can optimize his first order quantity q by expecting a future secondary transaction? More specifically, if both the producer and the distributor have expected a compensation opportunity, will they alter their ordering and shipping quantity decisions? If so, will they always benefit from the compensation opportunity? In the following we will study an extended business model to answer these questions.

# An Extended Business Model with Compensation

The extended model is described as follows: Given a wholesale price of w, the distributor first places a primary order requesting q units of products, considering the possibility of obtaining surplus inventory at a possibly discounted price, and the possibility of shortage supply from the producer. Then, the producer determines a shipping quantity Q(Q > q) and loads them onto the transportation vehicle. After the products arrive at the destination market and the observation of the surviving index,  $\theta$ , the following will occur: (i) if surviving quantity is less than or equal to q, then a supply shortage has occurred and the distributor gets all marketable inventory at wholesale price w; and (ii) if the surviving quantity is greater than q, then the producer satisfies the distributor's primary order of q and sells the surplus

 $(Q\theta - q)$  units at price  $\tilde{w}^*(Q\theta)$  to the distributor. Finally, the distributor sells the products to end customers at an optimized retail price.

Note that by ordering q units before transportation, the distributor is "pulling" some inventory from the producer; on the other hand, when the surviving quantity exceeds q, the producer is "pushing" the surplus inventory to the distributor. From the viewpoint of the entire supply chain, the extended model does not waste any of the surplus inventory and therefore might be more efficient than the pull or push model. However, compared with the pull model, will both parties *surely* be better off in the extended model? We will look into the question briefly.

We investigate the optimal decisions in the extended model in backwards order. First, knowing that she could sell all surplus inventory at price  $\tilde{w}^*(Q\theta)$ , the producer's expected profit as a function of her shipping quantity Q is

$$\Pi_{p}(Q \mid q) = \mathbf{E}\{w \min(q, Q\Theta) + \tilde{w}^{*}(Q\Theta)[Q\Theta - q]^{+} - (c_{1} + c_{2})Q\}$$
  
$$= wq - (c_{1} + c_{2})Q - w \int_{0}^{q/Q} (q - Qx)f(x)dx$$
  
$$+ \int_{q/Q}^{1} A(Qx - q)(Qx)^{-1/k}f(x)dx, \qquad (19)$$

given that the distributor's primary order quantity is q.

**Theorem 5:** In the extended business model, given that the distributor's primary order quantity is *q*:

(i) If q = 0, then the producer's optimal shipping quantity is

$$Q^{*}(q) = \left(\frac{1}{c_{1} + c_{2}} \times \frac{k - 1}{k} A \mathbf{E}\{\Theta^{1 - 1/k}\}\right)^{k};$$
(20)

(ii) Otherwise, the producer's optimal shipping quantity  $Q^*(q) = q/\theta(q)$ , where  $\theta(q)$  is dependent of q and must satisfy

$$c_1 + c_2 - w \int_0^\theta x f(x) dx$$
  
=  $\frac{A}{k} q^{-1/k} \theta \int_{\theta}^1 \left[ (k-1) \left(\frac{x}{\theta}\right)^{1-1/k} + \left(\frac{x}{\theta}\right)^{-1/k} \right] f(x) dx.$  (21)

**Proof:** To characterize the structure of the producer's profit, we take the first derivative of  $\prod_p (Q \mid q)$  with respect to Q:

$$\Pi'_{p}(Q \mid q) = -(c_{1} + c_{2}) + w \int_{0}^{q/Q} xf(x)dx + \frac{A}{kQ} \int_{q/Q}^{1} [(k-1)(Qx)^{1-1/k} + q(Qx)^{-1/k}]f(x)dx.$$
(22)

(i) If q = 0, the above derivative becomes:

$$\Pi'_p(Q \mid q) = -(c_1 + c_2) + \frac{k - 1}{k} A \int_0^1 Q^{-1/k} x^{1 - 1/k} f(x) dx,$$

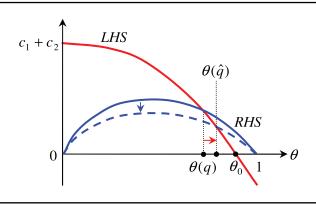


Figure 4: Illustration for the solution of Equation (21).

which is decreasing in Q. Therefore,  $\Pi_p(Q | q)$  is concave, and the optimal shipping quantity is uniquely determined by the first-order condition, from which we arrive at Equation (20).

(ii) If q > 0, unfortunately,  $\Pi'_p(Q | q)$  may not be monotonously decreasing, therefore,  $\Pi_p(Q | q)$  may not be concave or unimodal. As a result, it is possible that  $\Pi_p(Q | q)$  has multiple local maximizers. However, it is a necessary condition that any maximizer (including the global maximizer) must satisfy the first-order condition, from which we have  $Q^*(q) = q/\theta(q)$ , where  $\theta(q)$  is a solution of Equation (21). This completes the proof.

As Theorem 5 shows, unlike the pull and push models, in the extended model, the inventory-plus factor (when the distributor places a positive primary order) is no longer independent of the quantity ordered by the distributor. This is because her revenue from compensation by distributor is not proportional to q. From Equation (21), we clearly have

$$c_1+c_2>w\int_0^{\theta(q)}xf(x)\mathrm{d}x;$$

from which we have  $\theta(q) < \theta_0$  (recall Equation (5)). That is, with a secondary transaction opportunity, the producer prefers a higher inventory-plus factor because she has less risk in having surplus inventory.

As stated, when the distributor places a positive primary order (i.e., q > 0), the first-order Equation (21) is only a necessary condition, because the last item in  $\Pi'_p(Q | q)$  as presented in Equation (22) is not decreasing in Q for a general distribution of  $\Theta$  (therefore  $\Pi_p(Q | q)$  may not be concave). However, for some common distributions, including the uniform and exponential, the right-hand-side (RHS) of Equation (21) can easily be shown to be unimodal in  $\theta$  (see illustrative Figure 4), with values equal to zero for  $\theta = 0$  and  $\theta = 1$ ; whereas the left-hand-side (LHS) is strictly decreasing in  $\theta$ , with the values being positive and negative for  $\theta$ = 0 and  $\theta = 1$ , respectively (recall Assumption 2). Therefore, it is intuitive that Equation (21) has a unique solution. In such cases, when q increases (suppose qincreases to  $\hat{q}$ ), the RHS becomes flatter (see the dashed line in Figure 4) and the solution to Equation (21) increases as well (i.e.,  $\theta(\hat{q}) > \theta(q)$ ). This implies that the producer will choose a smaller inventory-plus factor when the distributor requests more products in the primary transaction.

Next, we turn to investigate the distributor's primary order quantity, knowing that the quantity shipped by the producer is  $Q^*(q)$ . It is seen that all the producer's surviving inventory flows to the distributor in the extended model, and the distributor's purchasing cost now consists of two parts: that paid at fixed cost w and that paid at uncertain cost  $\tilde{w}^*(\cdot)$ . Note that by allowing a secondary transaction opportunity, the distributor is likely to order less than the quantity in the pull model (even zero quantity). If  $q < (A/w)^k$  and the surviving quantity  $Q\theta \in (q, (A/w)^k)$ , then the compensation rate  $\tilde{w}^*(Q\theta) = A(Q\theta)^{-1/k}$  may be even higher than w.

We formulate the distributor's expected profit as a function of his primary order quantity q as:

$$\Pi_{d}(q) = \mathbf{E} \{ R_{d}^{*}(Q^{*}(q)\Theta) - w \min(q, Q^{*}(q)\Theta)^{+} - \tilde{w}^{*}(Q^{*}(q)\Theta)(Q^{*}(q)\Theta - q)^{+} \}.$$
(23)

If q = 0 (i.e., the distributor gambles solely on the secondary transaction to obtain inventories), the business model reduces to a form similar to the push model, except that the producer's wholesale price is no longer fixed at the pre-set level w; instead, it becomes flexible and random, which depends on the eventual surviving quantity  $Q\theta$ . By substituting Equation (20) into Equation (23), we arrive at the distributor's profit

$$\Pi_d(0) = \frac{1}{k-1} (c_1 + c_2)^{1-k} \left(\frac{k-1}{k}\right)^{k-1} (A\mathbf{E}\{\Theta^{1-1/k}\})^k.$$
(24)

If q > 0, we could substitute  $\theta(q)$  obtained from Equation (21) into Equation (23) and then optimize  $\Pi_d(q)$ . However,  $\theta(q)$  does not have a closed-form formulation and the profit function in Equation (23) becomes more complicated than the previous form Equation (8). Therefore it is difficult to characterize the structure of  $\Pi_d(q)$ , although the optimal order quantity q must satisfy the first-order condition.

Let the distributor's optimal primary order quantity be

$$q^* = \arg \max_{q \ge 0} \{ \Pi_d(q) \}.$$

First, we must have  $\Pi_d(q^*) > \Pi_d^{c*}$ ; the distributor should always benefit from the secondary transaction. This can be justified from the following. Suppose in the extended model, the distributor orders  $q_c^*$ , the optimal quantity in the pull model. Then, the shipping quantity chosen by the producer should be greater than that in the pull model (recall that  $\theta(q_c^*) < \theta_0$ ). As a result, the supply becomes more reliable and the distributor gains extra profit from any surplus inventory, and intuitively, the overall expected profit of the distributor increases.

However, on the side of the producer, one cannot guarantee that the producer will always achieve a higher profit than that in the pull model. On the contrary, she may even be worse off if the distributor chooses a rather low primary order quantity (e.g.,  $q < (A/w)^k$ ). This is because, by allowing compensation, the distributor may transfer more transportation risks to the producer by ordering less (he knows that

the producer will still ship a sufficiently large amount of products). Eventually, the producer may suffer from a profit decrease in the extended model.

#### A Fixed Inventory-plus Factor (FIPF) Strategy

As analyzed, by offering a secondary transaction at compensation rate  $\tilde{w}^*(\cdot)$  on the surplus inventory, the producer may not benefit at all because such an offer distorts the primary ordering decisions of the distributor. Therefore, the producer may be reluctant to commit on a secondary transaction without any other clauses.

Recall that in both the pull and push models, the optimal inventory-plus factors are the same,  $1/\theta_0$ , which is independent of the order quantity of the distributor. This motivated us to come up with the FIPF strategy for the producer. That is, while committing to a secondary transaction opportunity on the possible surplus inventory, the producer commits to ship  $1/\theta_0$  times the quantity ordered by the distributor. By ensuring this, the producer may prevent the distributor from ordering less inventory. In this subsection, we investigate the performance of both parties under the FIPF strategy.

In this model, the producer does not need to decide on her shipping quantities. The sequence of events is almost the same as that in the extended model, except that the producer's shipping quantity is given directly by  $Q = q/\theta_0$ . Given the response of the producer, the distributor's profit as a function of his order quantity is given by

$$\Pi_{d}(q) = \mathbf{E} \left\{ R_{d}^{*} \left( \frac{q}{\theta_{0}} \Theta \right) - w \min \left( q, \frac{q}{\theta_{0}} \Theta \right) - \tilde{w}^{*} \left( \frac{q}{\theta_{0}} \Theta \right) \left( \frac{q}{\theta_{0}} \Theta - q \right)^{+} \right\}$$
$$= -wq + \frac{kA}{k-1} \int_{0}^{1} \left( \frac{q}{\theta_{0}} x \right)^{1-1/k} f(x) dx$$
$$+ \int_{0}^{\theta_{0}} w \left( q - \frac{q}{\theta_{0}} x \right) f(x) dx - A \int_{\theta_{0}}^{1} \left( \frac{q}{\theta_{0}} x - q \right) \left( \frac{q}{\theta_{0}} x \right)^{-1/k} f(x) dx.$$

The first derivative is

$$\Pi'_d(q) = Aq^{-1/k} \left[ \int_0^1 \left(\frac{x}{\theta_0}\right)^{1-1/k} f(x) dx - \frac{k-1}{k} \int_{\theta_0}^1 \left(\frac{x}{\theta_0} - 1\right) \left(\frac{x}{\theta_0}\right)^{-1/k} f(x) dx \right] \\ + \int_0^{\theta_0} w \left(1 - \frac{x}{\theta_0}\right) f(x) dx - w,$$

which is decreasing in q. Therefore, the profit function is concave, and the optimal order quantity is uniquely determined by the first-order condition. We summarize the optimal decisions under the FIPF strategy in the following theorem.

**Theorem 6:** Under the FIPF strategy, the distributor's optimal order quantity  $q^*$  is

$$q^* = \left(\frac{A}{w}\tilde{\Omega}(\theta_0)\right)^k,\tag{25}$$

where function  $\tilde{\Omega}(\cdot)$  is defined as

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$$\tilde{\Omega}(\theta) := \frac{\int_0^1 \left(\frac{x}{\theta}\right)^{1-1/k} f(x) dx - \frac{k-1}{k} \int_{\theta}^1 \left(\frac{x}{\theta} - 1\right) \left(\frac{x}{\theta}\right)^{-1/k} f(x) dx}{1 - \int_0^{\theta} \left(1 - \frac{x}{\theta}\right) f(x) dx}, \quad (26)$$

and the producer's shipping quantity is  $Q^* = q^*/\theta_0$ .

To compare  $q^*$  with  $q_c^*$ , we have

$$\tilde{\Omega}(\theta) - \Omega(\theta) = \frac{\int_{\theta}^{1} \left[ \frac{1}{k} \left( \frac{x}{\theta} \right)^{1-1/k} + \frac{k-1}{k} \left( \frac{x}{\theta} \right)^{-1/k} - 1 \right] f(x) \mathrm{d}x}{1 - \int_{0}^{\theta} \left( 1 - \frac{x}{\theta} \right) f(x) \mathrm{d}x} > 0.$$

The inequality holds because it is easy to show that a function

$$\varphi(y) := \frac{1}{k} y^{1-1/k} + \frac{k-1}{k} y^{-1/k} - 1$$

is strictly increasing in  $y \in [1, +\infty)$ ; and as a result, for  $\forall y \ge 1$ ,  $\varphi(y) \ge \varphi(1) = 0$ .

By comparing Equation (25) with Equation (9), we immediately arrive at  $q^* \ge q_c^*$ . That is, under the FIPF strategy, the producer will eventually induce the distributor to order more products (compared with the pull model). By doing so, (i) the distributor's profit is improved, because  $\Pi_d(q^*) \ge \Pi_d(q_c^*) > \Pi_d^c(q_c^*) = \Pi_d^{c*}$ ; and (ii) the producer's profit, which is readily shown to be an increasing function of the distributor's order quantity, denoted as  $\Pi_p(q^*)$ , also improves, as  $\Pi_p(q^*) \ge \Pi_p(q_c^*) > \Pi_p^c(q_c^*) = \Pi_p^{c*}$ . Therefore, the proposed FIPF strategy is incentive compatible and is a Pareto improvement over the pull model; it improves the respective performance of the two supply chain members by inducing the distributor to increase his primary order quantity.

Of course, the practical adoption of the FIPF strategy will require the supply chain members to share their information and trust in one another. In particular, if the producer deliberately alters the actual inventory plus factor in hope of improving her own profit, she may harm the profitability of the distributor. Suppose the producer and distributor have formed a trust-based strategic alliance while adopting the FIPF strategy, two natural questions arise: (i) What is the magnitude of potential profit improvement by adopting the FIPF strategy? and (ii) Which party will benefit more from the FIPF strategy? We seek answers to these questions by conducting some numerical studies in the next section.

#### NUMERICAL STUDIES

In this section, we report the results of numerical experiments designed to gain insight into the impact of some key parameters, including uncertainties associated with product deterioration during the long-distance transportation and the priceelasticity of end-customer demand. Besides comparing the pull with the push models, we seek to evaluate the magnitude of profit improvement (over the pull model) by adopting the proposed FIPF strategy.

To lighten the computational effort, we assume the surviving factor  $\Theta$  follows a uniform distribution over the interval  $[\mu - \sigma, \mu + \sigma]$  with  $0 < \sigma \le \min(\mu, 1 - \mu)$ .

The parameter  $\sigma$  measures the deviation of  $\Theta$ : for given  $\mu$ ,  $\Theta$  has higher deviation when  $\sigma$  is large. Note that the impact of the uncertainty and size associated with the market demand are both sealed in the constant *A*; therefore, without loss of generality, we normalize A = 10. The values of other base parameters are the following:  $c_1 + c_2 = 1$ ,  $\mu = 0.6$ ,  $\sigma = 0.25$ , w = 3.5, and k = 1.8. We alter the values of  $\mu$ ,  $\sigma$ , and *k*, respectively, in each group of experiments.

First, a sample of the numerical results with different values of  $\mu$  and  $\sigma$  are reported in Table 2. We evaluate the optimal shipping quantity of the producer and the optimal expected profits for both supply chain members under the pull and push models and the FIPF strategy, respectively. Using the pull model as a benchmark, we measure the relative profit loss/improvement of the push model and the FIPF strategy. Table 2 reveals some interesting patterns not readily obtainable from the analytical formulation.

- (i) For all three scenarios, the optimal shipping quantity of the producer is decreasing in the mean value (for given deviation) and increasing in the deviation (for given mean value) of Θ. This is consistent with our intuition: the producer should ship more when the product is more perishable, and she should also ship more to hedge against the higher deterioration risk during transportation. As a result, both the producer and distributor's profits are decreasing in μ and σ, as expected.
- (ii) Compared with the pull model, it seems that the adoption of the push model has a more significant impact on the producer's performance. As can be seen, the producer's profit reduction is quite high (the average reduction is more than 10%), whereas the distributor's profit reduction is rather low (mostly less than 1%). This implies that generally, the producer has more motivation to choose the pull model. Moreover, it is shown that the reduction in the profit of both parties is more significant when the product is more perishable (i.e., when  $\mu$  is small) and when the deterioration has higher uncertainty (i.e., when  $\sigma$  is large).
- (iii) Compared with the pull model, it seems that the producer benefits more from adopting the FIPF strategy. As can be seen, the producer's profit increases significantly (by more than 10% for most cases), whereas the distributor's profit only increases slightly (all below 1%). The results suggest that the FIPF is an efficient strategy that the producer should try to implement. Moreover, it is shown that both parties gain more from the FIPF strategy when the product is less perishable (i.e., when  $\mu$  is large) and when the deterioration has higher uncertainty (i.e., when  $\sigma$  is large).

Next, we alter the value of the price elasticity, k, and report the numerical results in Table 3. As was shown, for the three scenarios, the producer will ship more products when customer demand is more price sensitive (i.e., when k is large). Another interesting phenomenon is that the producer's expected profit is increasing, whereas the distributor's profit is decreasing in k; this implies that the producer's relative power in the supply chain is stronger for a more price-sensitive market. Similar to the findings from Table 2, compared with the pull model, the adoption of a push model or an FIPF strategy mainly affects the performance of

		-	CIF model				PPS model				H	FIPF strategy	y	
ή	α	${oldsymbol{\mathcal{Q}}_{c}^{*}}$	$\Pi_p^{c*}$	$\Pi_d^{c*}$	${oldsymbol{\mathcal{Q}}_{s}^{*}}$	$\Pi_p^{s*}$	Loss	$\Pi_d^{s*}$	Loss	$Q^{*}$	$\Pi_p^*$	Gain	$\Pi_d^*$	Gain
0.45	0.25	14.97	7.73	28.38	11.59	5.99	-22.55%	28.02	-1.29%	14.99	8.44	9.20%	28.41	0.10%
0.50	0.25	13.75	9.08	28.55	11.21	7.41	-18.47%	28.31	-0.84%	13.78	10.10	11.21%	28.59	0.16%
0.55	0.25	12.72	10.21	28.66	10.80	8.66	-15.14%	28.50	-0.55%	12.75	11.52	12.83%	28.73	0.23%
0.60	0.25	11.83	11.17	28.74	10.36	9.78	-12.44%	28.64	-0.37%	11.86	12.74	14.07%	28.83	0.29%
0.65	0.25	11.05	12.00	28.80	9.91	10.76	-10.26%	28.73	-0.25%	11.08	13.79	14.99%	28.90	0.35%
0.70	0.25	10.35	12.72	28.84	9.47	11.64	-8.51%	28.79	-0.17%	10.39	14.71	15.64%	28.95	0.39%
0.60	0.10	11.46	11.68	28.93	10.96	11.17	-4.37%	28.92	-0.04%	11.47	12.55	7.46%	28.95	0.08%
0.60	0.15	11.63	11.49	28.89	10.82	10.69	-6.92%	28.86	-0.11%	11.64	12.66	10.24%	28.94	0.15%
0.60	0.20	11.75	11.32	28.83	10.62	10.23	-9.64%	28.77	-0.22%	11.77	12.72	12.42%	28.90	0.23%
0.60	0.25	11.83	11.17	28.74	10.36	9.78	-12.44%	28.64	-0.37%	11.86	12.74	14.07%	28.83	0.29%
0.60	0.30	11.87	11.03	28.62	10.06	9.34	-15.26%	28.46	-0.56%	11.91	12.71	15.25%	28.72	0.34%
0.60	0.35	11.87	10.89	28.45	9.73	8.93	-18.01%	28.22	-0.80%	11.91	12.64	16.05%	28.56	0.38%

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Table 2:

		CIF model	_			PPS model					FIPF strategy		
k	${oldsymbol{\mathcal{Q}}_{c}^{*}}$	$\Pi_p^{c*}$	$\Pi_d^{c*}$	$Q^*_s$	$\Pi_p^{s*}$	Loss	$\Pi_d^{s*}$	Loss	$\delta^*$	$\Pi_p^*$	Gain	$\Pi_d^*$	Gain
1.3	7.02	6.63	45.50	6.13	5.78	-12.74%	45.41	-0.20%	7.03	7.49	12.95%	45.57	0.14%
1.4	7.79	7.36	37.87	6.81	6.42	-12.66%	37.78	-0.25%	7.81	8.33	13.23%	37.94	0.18%
1.5	8.65	8.16	33.62	7.56	7.14	-12.59%	33.53	-0.28%	8.67	9.26	13.48%	33.69	0.21%
1.6	9.60	9.06	31.10	8.40	7.93	-12.54%	31.00	-0.32%	9.62	10.30	13.70%	31.18	0.24%
1.7	10.66	10.06	29.59	9.32	8.80	-12.49%	29.49	-0.34%	10.68	11.46	13.89%	29.67	0.27%
1.8	11.83	11.17	28.74	10.36	9.78	-12.44%	28.64	-0.37%	11.86	12.74	14.07%	28.83	0.29%
1.9	13.13	12.40	28.37	11.50	10.86	-12.40%	28.26	-0.39%	13.17	14.16	14.23%	28.45	0.31%
2.0	14.58	13.76	28.34	12.78	12.06	-12.36%	28.23	-0.41%	14.63	15.74	14.38%	28.44	0.33%
2.1	16.19	15.28	28.61	14.19	13.40	-12.33%	28.49	-0.42%	16.24	17.50	14.51%	28.71	0.35%
2.2	17.97	16.97	29.12	15.76	14.88	-12.30%	28.99	-0.44%	18.04	19.45	14.63%	29.22	0.37%
2.3	19.96	18.84	29.84	17.51	16.53	-12.28%	29.71	-0.45%	20.03	21.62	14.74%	29.96	0.38%

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Table 3:

the producer, and only has a slight impact on the distributor's profit. Moreover, the producer loses less (in the push model) and gains more (under the FIPF strategy) when the price-elasticity is large; in contrast, the distributor loses more (in the push model) and gains more (under the FIPF strategy) when the price-elasticity is large.

# **CONCLUDING REMARKS**

Fresh-product supply chains involving long distance transportation have become increasingly common in international and domestic markets. Compared with the management of conventional supply chains, the highly perishable nature of products during transportation creates extra challenges in matching uncertain supply with uncertain demand for producers and distributors in the supply chain. Depending on the model of doing business, the product transportation risk has a different impact on the supply chain members. In this article, we conducted an extensive comparative study of different business models by considering a supply chain consisting of a single producer and a single distributor.

Specifically, by focusing on the scenario in which the producer is responsible for the product transportation, we studied two variants of business models that exist in practice. After an in-depth investigation of the optimal shipping quantity, order quantity, and retail price decisions for the pull and push models, we show that both supply chain members will achieve better performance by adopting the pull model. This suggests that firms involved in the fresh-product supply chain switch from the push model to the pull model. Considering that the producer may suffer from having surplus inventory, we propose a compensation opportunity to deal with any surplus inventory at a possibly discounted wholesale price. Although the secondary transaction benefits both supply chain members by offering such an opportunity before the distributor places his primary order, the producer is likely to be worse off because the secondary transaction may motivate the distributor to order less and therefore increases the risk faced by the producer. We then suggest the FIPF strategy, under which the producer ships  $1/\theta_0$  times the quantity ordered and the distributor compensates the producer for any surplus inventory that would otherwise be wasted. We show that both parties will be better off under the FIPF strategy. Finally, numerical experiments are conducted to evaluate the magnitude of profit improvement by adopting the FIPF strategy. The major finding is that the FIPF strategy benefits the producer much more significantly than the distributor, especially when the product is less perishable, when the perishability has higher uncertainty, and/or when the end-customers are more price-sensitive.

Considering the uncertain product decay during transportation and distribution processes provides vast opportunities for future research: (i) Firstly, in this study we only compare business models for the scenario in which the producer is in charge of the product transportation. It would be interesting to compare the business models studied in this article with other potential business models (e.g., the free-on-board model, for example) by considering the wholesale price as a decision variable. (ii) The FIPF strategy proposed in this article is only a Pareto improvement over the pure pull model; naturally, whether we can design suitable mechanisms to induce the two parties to act in a coordinated way, so that

the maximal performance of the supply chain can be achieved, remains an open problem. (iii) Because the long-distance transportation is time-consuming, it is quite likely that the producer and/or distributor may use the updated information regarding market demand to make better decisions in the second period. Therefore, to consider information updating, for example, by assuming full and asymmetric information between supply chain members (Cachon & Lariviere, 2001), is another line of possible future research toward fresh-product supply chains. (iv) We have considered the simplest scenario with only one producer and one distributor. It will be interesting to study scenarios with multiple suppliers and/or multiple distributors. For example, by delivering products to multiple distributors, the producer may hedge against transportation risk from the potential inventory pooling effect. This opens a new and important direction for the future research toward fresh-product supply chain management. (v) Finally, as suggested by Blackburn and Scudder (2009) and Cai et al. (2010), an important dimension of the management of fresh product supply chains is to reduce losses from product perishability by, for example, shortening the transportation lead time and therefore reducing transportation delays, implementing temperature control, adopting chemical treatments, and improving cold storage capabilities. Therefore, it will be interesting to extend our models to the case incorporating freshness-keeping effort decisions.

## REFERENCES

- Anonymous. (2003). Fruit waste utilisation, accessed December 23, 2011, available at http://practicalaction.org/practicalanswers/product\_info. php?products\_id=96.
- Anonymous. (2010). Food spoilage rate still high in China, accessed December 23, 2011, available at http://www.supplychains.com/en/art/3811/.
- Anupindi, A., & Akella, A. (1993). Diversification under supply uncertainty. *Management Science*, 39(8), 994–963.
- Blackburn, J. D., & Scudder, G. D. (2009). Supply chain strategies for perishable products: The case of fresh produce. *Production and Operations Management*, 18(2), 129–137.
- Bolton, J. M., & Liu, W. B. (2006). Creating an effective China "cold supply chain"—Current status, challenges and implementation considerations. Accenture report, accessed December 23, 2011, available at http://www.procurementleaders.com/8201/23171/SCMChinaColdChinaFINAL110306. pdf
- Cachon, G.P. (2004). The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts. *Management Science*, 50(2), 222–238.
- Cachon, G. P., & Lariviere, M. A. (2001). Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Science*, 47(5), 629–646.

- Cai, X. Q., Chen, J., Xiao, Y. B., & Xu, X. L. (2010). Optimization and coordination of fresh product supply chains with freshness keeping efforts. *Production and Operations Management*, 19(3), 261–278.
- Cattani, K., Perdikaki, O., & Marucheck, A. (2007). The perishability of online grocers. *Decision Sciences*, *38*(2), 329–355.
- Chen, J. F., Yao, D. D., & Zheng, S. H. (2001). Optimal replenishment and rework with multiple unreliable supply sources. *Operations Research*, 49(3), 430–443.
- Ciarallo, F. W., Akella, R., & Morton, T. E. (1994). A Periodic review, production planning model with uncertain capacity and uncertain demand—Optimality of extended myopic policies. *Management Science*, 40(3), 320–332.
- Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. *IIE Transactions*, 5(4), 323–326.
- Dada, M., Petruzzi, N. C., & Schwarz, L. B. (2007). A newsvendor procurement problem with unreliable suppliers. *Manufacturing & Service Operations Management*, 9(1), 9–32.
- Ferguson, M., & Ketzenberg, M. E. (2006). Information sharing to improve retail product freshness of perishables. *Production and Operations Management*, 15(1), 57–73.
- Ferguson, M., & Koenigsberg, O. (2007). How should a firm manage deteriorating inventory? *Production and Operations Management*, *16*(3), 306–321.
- Gerchak, Y. (1996). A newsvendor with multiple, unreliable, all-or-nothing suppliers. Working article, University of Waterloo, Waterloo, Ontario.
- Ghare, P. M., & Schrader, G. F. (1963). A model for an exponentially decaying inventory. *The Journal of Industrial Engineering*, 14, 238–243.
- Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operations Research*, 134(1), 1–16.
- Henig, M., & Gerchak, Y. (1990). The structure of periodic review policies in the presence of random yield. *Operations Research*, 38(4), 634–643.
- Lariviere, M., & Porteus, E. (2001). Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management*, 3(4), 293–305.
- Leung, E. (2008). Supply chain analysis: Maintaining product freshness, accessed December 23, 2011, available at http://www.asiafoodjournal.com/article-5171-supplychainanalysismaintainingproductfreshness-Asia.html.
- Mookherjee, R., & Friesz, T. L. (2008). Pricing, allocation, and overbooking in dynamic service network competition when demand is uncertain. *Production* and Operations Management, 17(4), 455–474.
- Nahmias, S. (1982). Perishable inventory theory: A review. *Operations Research*, *30*(4), 680–708.
- Petruzzi, N. C., & Dada, M. (1999). Pricing and the newsvendor problem: A review. Operations Research, 47(2), 183–194.

- Philip, G. C. (1974). A generalized EOQ model for items with Weibull distribution deterioration. *IIE Transactions*, 6(2), 159–162.
- Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, 42(1), 27–37.
- Tadikamalla, P. R. (1978). An EOQ model for items with Gamma distribution deterioration. *IIE Transactions*, 10(1), 100–103.
- Wang, Y. (2006). Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Operations Research*, 54(6), 1110–1127.
- Wang, Y., & Gerchak, Y. (1996). Periodic review production models with variable capacity, random yield, and uncertain demand. *Management Science*, 42(1), 130–137.
- Wang, Y., Jiang, L., & Shen, Z. (2004). Channel performance under consignment contract with revenue sharing. *Management Science*, 50(1), 34–47.
- Whitin, T. M. (1957). *Theory of inventory management*. Princeton, NJ: Princeton University Press.
- Wilson, L. G., Boyette, M. D., & Estes, E. A. (1995). Postharvest handling and cooling of fresh fruits, vegetables, and flowers for small farms. Horticulture Information Leaflets, NC State University, accessed December 23, 2011, available at http://www.ces.ncsu.edu/depts/hort/hil/hil-800.html.
- Yano, C. A., & Lee, H. L. (1995). Lot sizing with random yields: A review. *Operations Research*, 43(2), 311–334.
- Zuurbier, P. J. P. (1999). Supply chain management in the fresh produce industry: A mile to go. *Journal of Food Distribution Research*, *30*(1), 20–30.

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